Beyond Plurality: Truth-Bias in Binary Scoring Rules

(Extended Abstract)

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ABSTRACT

It is well known that standard game-theoretic approaches to voting mechanisms lead to a multitude of Nash Equilibria (NE), many of which are counter-intuitive. We focus on truth-biased voters, a model recently proposed to avoid such issues. The model introduces an incentive for voters to be truthful when their vote is not pivotal. This is a powerful refinement, and recent simulations reveal that the surviving equilibria tend to have desirable properties.

However, truth-bias has been studied only within the context of plurality elections, which is an extreme example of k-approval rules with $k = 1$. We undertake an equilibrium analysis of the complete range of k-approval rules (except veto).

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multi-Agent Systems

General Terms

Algorithms, Theory, Economics

Keywords

Truth bias; Nash equilibrium; Social choice; Plurality; Veto

1. INTRODUCTION

Most voting games contain an enormous amount of Nash Equilibria (NE), with even small games counting hundreds of thousands. Furthermore, many NEs are formed by votes which will not occur in the real world (e.g., for most voting rules, if all voters rank the same candidate last, the case where all voters vote for this least favorite option is a NE).

There have been many modeling approaches towards eliminating the multitude of Nash equilibria in voting games. Some are based on introducing uncertainty, either regarding the support of each candidate [8, 11], or about the reliability of counting procedures [7]. Other research suggests changing the temporal structure of the game [13, 1]. A different approach is the notion of lazy voting [1], where the utility function is changed so that non-pivotal voters have a slight preference to abstain.

Another way to refine the set of equilibria is to stick to the basic game-theoretic models, but study equilibria that are reachable by iterative voting procedures. The iterative voting model was introduced in [6] and later expanded by [5]. Those papers followed research into iterative and dynamic mechanisms, summarised in [3].

We focus on a different model than the approaches above for refining the set of equilibria, namely truth bias. The notion of adding truth bias to games was introduced (for a specific case) in [4], and was proposed for a specific voting rule (with limited results) in [2]. A more robust model was suggested in [12], which introduced the general framework, and contained various empirical results. The theoretical side of that work was enhanced in [9]. More recent work has also attempted to relate this line of research to iterative voting [10], but this again is solely with respect to plurality.

2. DEFINITIONS AND NOTATION

We adopt the notation of Obraztsova et al [9]. In particular, we suppose that voters have a slight preference for voting truthfully when they cannot unilaterally affect the outcome of the election. This *truth bias* is captured by inserting a small extra payoff when the voter votes truthfully. In this paper we tackle the complexity of the following two problems under the truth-bias assumption.

Definition 1 ($\exists NE$). *An instance of the $\exists NE$ problem is determined by a preference profile $\mathbf{a}$, and will be denoted by $\exists NE(\mathbf{a})$. The profile $\mathbf{a}$ indicates the true preferences of the voters. Given $\mathbf{a}$, $\exists NE(\mathbf{a})$ is a "yes" instance $\iff$ the corresponding game, with truth-biased voters, admits at least one Nash equilibrium.*

Definition 2 ($WinNE$). *An instance of the $WinNE$ problem is determined by a preference profile $\mathbf{a}$, and a candidate $w \in C$, denoted by $WinNE(w, \mathbf{a})$. It is a "yes" instance $\iff$ the corresponding game, with truth-biased voters, admits at least one Nash equilibrium with w as the winner.*

Finally, we will utilise the following definitions of *runner-up* and *threshold* candidates.

Definition 3. *In a profile $\mathbf{b}$, where the winner is $\mathcal{F}(\mathbf{b})$, a runner-up candidate is a candidate $c \in C$, for which one of the following conditions hold:*

- $\mathrm{sc}(c, \mathbf{b}) = \mathrm{sc}(\mathcal{F}(\mathbf{b}), \mathbf{b})$*, and* $\mathcal{F}(\mathbf{b}) \succ c$ *in tie-breaking,*

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- $\mathrm{sc}(c, \mathbf{b}) = \mathrm{sc}(\mathcal{F}(\mathbf{b}), \mathbf{b}) - 1$, *and* $c \succ \mathcal{F}(\mathbf{b})$ *in tie-breaking.*

Given a voting profile $\mathbf{b}$*, a threshold candidate* c *is a runner-up candidate for which one of the following holds:*

- c *is the maximal element of* $\mathbf{R}_1$ *w.r.t. the tie-breaking order, if* $\mathbf{R}_1 \neq \emptyset$*,*
- c *is the maximal element of* $\mathbf{R}_2$ *w.r.t. the tie-breaking order, if* $\mathbf{R}_1 = \emptyset$*.*

3. TRUTH-BIAS UNDER K-APPROVAL

For k-approval with $k \geq 1$, voters "approve" of the first k candidates in their submitted ballot; hence each such candidate receives one point from that voter. Clearly the NP-hardness results for plurality, established in [9], continue to hold, since plurality is a special case of k-approval. However, the rest of the properties identified in [9], do not hold for the more general class of k-approval rules.

Let $\mathcal{A}_i$ (respectively, $\mathcal{B}_i$) be the set of approved candidates in a profile a_i (respectively, b_i), and let the terms "votes in favor" and "votes against" mean the following. Let $\mathbf{a}$ be the truthful profile, and let $\mathbf{b}$ be the submitted profile. A voter i **votes in favor** of a candidate c_j, if $c_j \notin \mathcal{A}_i$ and $c_j \in \mathcal{B}_i$. Similarly, i **votes against** c_j, if $c_j \in \mathcal{A}_i$ and $c_j \notin \mathcal{B}_i$.

Lemma 1. *Given a Nash Equilibrium (NE) profile* $\mathbf{b}^{NE} \neq \mathbf{a}$*, for every non-truhful voter i, exactly one, but never both, of the following conditions hold: a) i votes in favor of the winner, b) i votes against some* $r \in R_1 \cup R_2$*.*

Note that for plurality, it was established in [9] that it is case (a) that holds, and never case (b). For k-approval, and arbitrary values for k, it can be either of the two cases. Next, we establish that a threshold candidate always exists.

Proposition 1. *For every equilibrium* $\mathbf{b}^{NE} \neq \mathbf{a}$*, a threshold candidate always exists.*

Unlike plurality, in the k-approval case, it is possible that in a non-truthful equilibrium, neither the winner nor the threshold candidate will maintain their truthful profile score. This is demonstrated in the following example.

Example 1. *Consider the following two profiles using 2-approval, with the tie-breaking order given by the sequence* $a \succ b \succ c \succ d \succ e$*. The truthful profile is:*

- $a \succ b \succ c \succ d \succ e$
- $e \succ d \succ a \succ c \succ b$
- *2 voters with preference* $d \succ b \succ a \succ c \succ e$
- *2 voters with preference* $a \succ d \succ b \succ c \succ e$
- $e \succ c \succ a \succ b \succ d$

The equilibrium profile changes the last but one voter (i.e., one out of the two identical voters with preference $a \succ d \succ b \succ c \succ e$*), as well as the last two voters to:*

$a \succ e \succ b \succ c \succ d$ *and* $e \succ a \succ c \succ b \succ d$.

In this example, the score of the winning candidate (more specifically, candidate a*) in the equilibrium profile is higher than in the truthful profile. On the other hand, the score of the threshold candidate (in this example* d*) decreases in the equilibrium compared to the truthful profile score.*

Finally, regarding the winner's score in a Nash equilibrium, we prove that it cannot fluctuate and go up or down as the score of the threshold candidate, but instead it is bounded, according to the following proposition.

Proposition 2. *Let* $w = \mathcal{F}(\mathbf{b}^{NE})$ *for a Nash equilibrium* $\mathbf{b}^{NE} \neq \mathbf{a}$*. Then* $\mathrm{sc}(w, \mathbf{b}^{NE}) \geq \mathrm{sc}(w, \mathbf{a})$.

Table 1: Results w.r.t. NE complexity and features

Conditions (w.r.t. NE)	Plurality	k-approval
$WinnerNE(w, \mathbf{a})$	NP-hard	NP-hard
Winner score may increase	Yes	Yes
Winner score may decrease	No	No
Runner-up score may increase	No	Yes
Runner-up score may decrease	No	Yes

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